Name: ________________________ Class: ___________________ Date: __________ ID: A

Algebra II Final Exam Semester II Practice Test

1. (10 points) A bacteria population starts at 2,032 and decreases at about 15% per day. Write a function representing the number of bacteria present each day. Graph the function. After how many days will there be fewer than 321 bacteria?

2. (10 points) A initial investment of $10,000 grows at 11% per year. What function represents the value of the investment after t years?

3. (10 points) A clerk needs to price a cashmere sweater returned by a customer. The customer paid a total of $222.36 that included a gift wrapping charge of $4 and a 9% sales tax. What price should the clerk mark on the tag?

4. (10 points) A wildlife biologist in Nova Scotia is testing the pH of stream water. She hopes that the pH of the water is greater than 5.5 so that Atlantic Salmon returning this year to their natal streams will be able to reproduce. The hydrogen ion concentration of the water is 0.0000019 moles per liter. To the nearest tenth, what is the pH of the water? (Recall that $\text{pH} = -\log\left[\text{H}^+\right]$.)

5. (10 points) Express $\log_3 6 + \log_3 4.5$ as a single logarithm. Simplify, if possible.

6. (10 points) Express $\log_2 64 - \log_2 4$ as a single logarithm. Simplify, if possible.

7. (10 points) The altitude of an aircraft, h, in miles, is given by $h = -\left(\frac{100}{9}\right)\log\frac{P}{B}$, where P = the outside air pressure, and B = the atmospheric pressure at sea level. Let B = 31 inches of mercury (in. Hg). What is the outside air pressure at a height of 3.6 miles? Round your answer to the nearest tenth.

8. (10 points) Simplify $\log_7 x^3 - \log_7 x$.

9. (10 points) Solve $8^{x+8} = 32^x$.

10. (10 points) The amount of money in a bank account can be expressed by the exponential equation $A = 300(1.005)^{12t}$ where A is the amount in dollars and t is the time in years. About how many years will it take for the amount in the account to be more than \$900?

11. (10 points) Solve $\log_5 x^{10} - \log_5 x^6 = 21$.

12. (10 points) Distance varies directly as time because as time increases, the distance traveled increases proportionally. The speed of sound in air is about 335 feet per second. How long would it take for sound to travel 11,725 feet?

13. (10 points) The volume V of a cylinder varies jointly with the height h and the radius squared r^2, and $V = 157.00$ cm^3 when $h = 2$ cm and $r^2 = 25$ cm^2. Find V when $h = 3$ cm and $r^2 = 36$ cm^2. Round your answer to the nearest hundredth.

14. (10 points) The number of lawns l that a volunteer can mow in a day varies inversely with the number of shrubs s that need to be pruned that day. If the volunteer can prune 6 shrubs and mow 8 lawns in one day, then how many lawns can be mowed if there are only 3 shrubs to be pruned?

15. (10 points) The pressure P of a gas varies inversely with the volume V of its container and directly with the temperature T. A certain gas has a pressure of 1.6 atmospheres with a volume of 14 liters and a temperature of 280 kelvins. If the gas is cooled to a temperature of 250 kelvins and the container is expanded to 16 liters, what will be the new pressure?

16. (10 points) Simplify $\dfrac{4y^3 - 8y}{y^2 - 2y}$. Identify any y-values for which the expression is undefined.

17. (10 points) Multiply $\dfrac{8x^4y^2}{3z^3} \cdot \dfrac{9xy^2z^6}{4y^4}$. Assume that all expressions are defined.

18. (10 points) Divide $\dfrac{5x^3}{3x^2y} \div \dfrac{25}{3y^9}$. Assume that all expressions are defined.

19. (10 points) Solve $\dfrac{x^2 + x - 30}{x - 5} = 11$. Check your answer.

20. (10 points) The area of a rectangle is equal to $x^2 + 10x + 16$ square units. If the length of the rectangle is equal to $x + 8$ units, what expression represents its width?

21. (10 points) Add $\dfrac{x+6}{x-7} + \dfrac{-12x - 59}{x^2 - 3x - 28}$.

22. (10 points) Subtract $\frac{2x^2 - 48}{x^2 - 16} - \frac{x+6}{x+4}$. Identify any x-values for which the expression is undefined.

23. (10 points) Solve the equation $x - 9 = -\frac{18}{x}$.

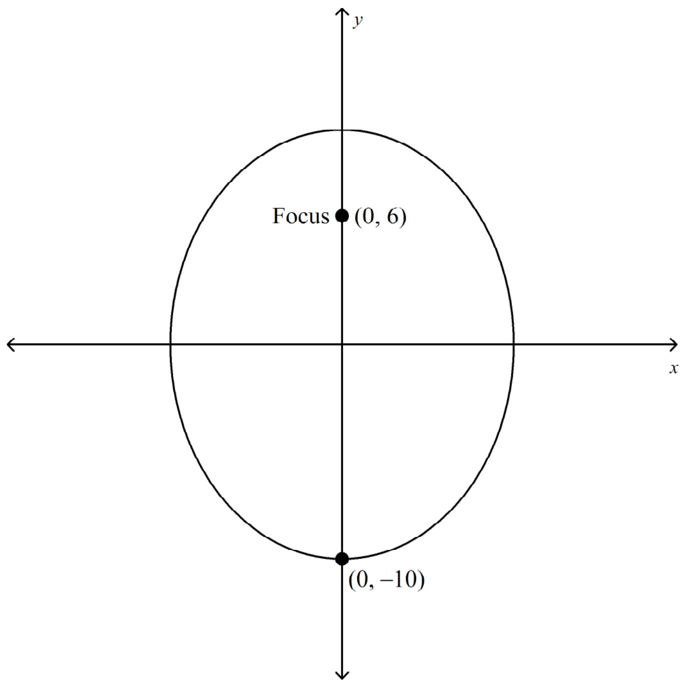
24. (10 points) Solve the equation $\frac{6x}{x-3} = \frac{4x+6}{x-3}$.

25. (10 points) Simplify the expression $\sqrt[4]{256z^{16}}$. Assume that all variables are positive.

26. (10 points) Simplify the expression $(27)^{\frac{1}{3}} \cdot (27)^{\frac{2}{3}}$.

27. (10 points) Solve the equation $-6 + \sqrt{x-5} = -2$.

28. (10 points) Write an equation in standard form for the ellipse shown with center (0, 0).



29. (10 points) Graph the ellipse $\dfrac{(x-6)^2}{100}+\dfrac{(y+5)^2}{64}=1$.

30. (10 points) The path that a satellite travels around Earth is an ellipse with Earth at one focus. The length of the major axis is about 16,000 km, and the length of the minor axis is about 12,000 km. Write an equation for the satellite's orbit.

31. (10 points) Write an equation in standard form for the hyperbola with center (0, 0), vertex (0, 6), and focus (0, 8).

32. (10 points) Find the vertices, co-vertices, and asymptotes of the hyperbola $\frac{(y-1)^2}{25} - \frac{(x+2)^2}{9} = 1$, and then graph.

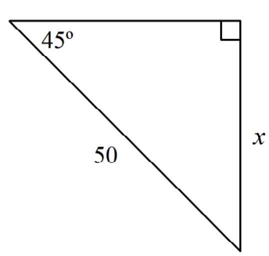
33. (10 points) Write the equation in standard form for the parabola with vertex (0,0) and the directrix $y = -14$.

34. (10 points) Find the vertex, value of p, axis of symmetry, focus, and directrix of the parabola $y - 2 = -\frac{1}{8}(x+2)^2$. Then, graph the parabola.

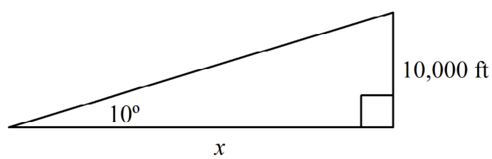
35. (10 points) Identify the conic section the equation $\frac{(x-2)^2}{3^2} + \frac{(y-4)^2}{7^2} = 1$ represents.

36. (10 points) Identify the conic section that the equation $4x^2 - 5xy - 5y^2 - 3x + 2y + 9 = 0$ represents.

37. (10 points) Use a trigonometric function to find the value of x.



38. (10 points) After takeoff from an airport, an airplane's angle of ascent is 10°. The airplane climbs to an altitude of 10,000 feet. At that point, what is the land distance between the airplane and the airport? Round your answer to the nearest foot.



39. (10 points) A surveyor whose eye level is 5 feet above the ground determines the angle of elevation to the top of an office building to be 41.7°. If the surveyor is standing 40 feet from the base of the building, what is the height of the building to the nearest foot?

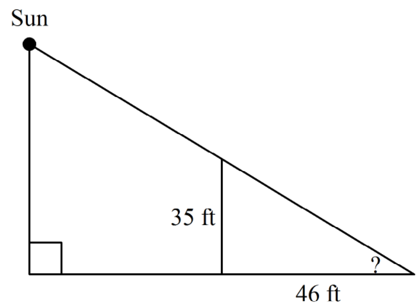
40. (10 points) Find the measure of the reference angle for $\theta = 142°$.

41. (10 points) $P(-7,-2)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.

42. (10 points) Use the unit circle to find the exact value of the trigonometric function cos 30°.

43. (10 points) Use a reference angle to find the value of sin 300º.

44. (10 points) A 35-foot telephone pole casts a 46-foot shadow on the ground while the sun is shining. To the nearest degree, what is the angle of elevation of the sun from the end of the shadow?



45. (10 points) Solve the equation $\sin\theta = 0.3$ to the nearest tenth. Use the restrictions $90° < \theta < 180°$.

46. (10 points) A triangle has a side with length 6 feet and another side with length 8 feet. The angle between the sides measures 73°. Find the area of the triangle. Round your answer to the nearest tenth.

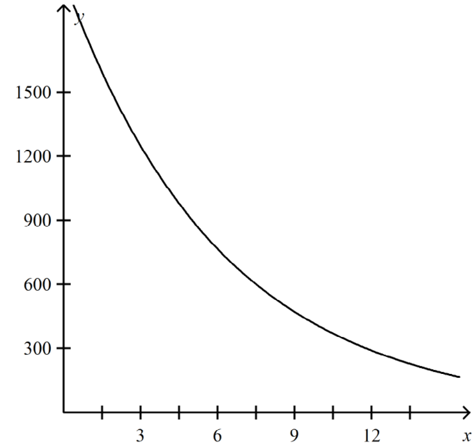
47. (10 points) Two airplanes leave the airport at the same time. One airplane flies due east at a speed of 300 miles per hour. The other airplane flies east-northeast at a speed of 350 miles per hour (the angle between the two directions is 22.5º). If the planes are at the same altitude, how far apart are they after 2 hours? Round your answer to the nearest mile.

48. (10 points) A building has a triangular floor with sides of 150, 200, and 280 feet. To the nearest square foot, what is the area of the floor of the building?

Algebra II Final Exam Semester II Practice Test
Answer Section

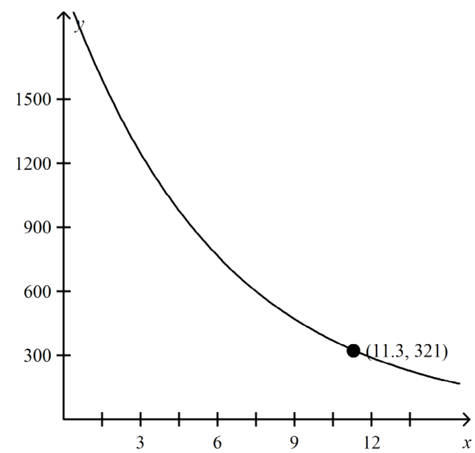
SHORT ANSWER

1. ANS:
$f(x) = 2032(0.85)^t$



After about 11.3 days, there will be fewer than 321 bacteria.

$f(x) = a(1-r)^t$	Write exponential decay function.
$f(x) = 2{,}032(1-0.15)^t$	Substitute 2032 for *a* and 015 for *r*.
$f(x) = 2{,}032(0.85)^t$	Simplify.



After about 11.3 days, there will be fewer than 321 bacteria.

PTS: 10 DIF: Average REF: Page 492 OBJ: 7-1.3 Application
NAT: 12.5.1.e TOP: 7-1 Exponential Functions Growth and Decay

2. ANS:

$f(t) = 10000(1.11)^t$

The investment follows an exponential growth of 11% per year with an initial value of $10,000. Using the formula $f(t) = P(1+r)^t$, substitute the given values.

$f(t) = 10000(1+11\%)^t$

$f(t) = 10000(1+0.11)^t$

$f(t) = 10000(1.11)^t$

PTS: 10 DIF: Advanced NAT: 12.5.1.e
TOP: 7-1 Exponential Functions Growth and Decay

3. ANS:

$200.00

$c = 1.09(p+4)$	Cost c is a function of price p.
$c = 1.09p + 4.36$	Distribute.
$c - 4.36 = 1.09p$	Subtract 4.36 from both sides.
$\frac{c-4.36}{1.09} = p$	Divide to isolate p.
$\frac{222.36-4.36}{1.09} = p$	Substitute 222.36 for c.
$200 = p$	Simplify.

The clerk should mark the price on the tag as $200.00.

PTS: 10 DIF: Average REF: Page 500 OBJ: 7-2.5 Application
TOP: 7-2 Inverses of Relations and Functions

4. ANS:

5.7

$\text{pH} = -\log\left[\text{H}^+\right]$

$\text{pH} = -\log(0.0000019)$ Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the [LOG] key.

The stream water has a pH of about 5.7. Atlantic Salmon returning to their natal streams will be able to reproduce this year!

PTS: 10 DIF: Average REF: Page 508 OBJ: 7-3.5 Application
TOP: 7-3 Logarithmic Functions

5. ANS:
   3

$\log_3 6 + \log_3 4.5 = \log_3 27$	To add the numbers, multiply the logarithms
3	Think. What exponent on base 3 gives 27? $3^3 = 27$

   PTS: 10 DIF: Basic REF: Page 514 OBJ: 7-4.1 Adding Logarithms
   TOP: 7-4 Properties of Logarithms

6. ANS:
   4
   To subtract the logarithms divide the numbers.
   $\log_2 64 - \log_2 4 = \log_2(\frac{64}{4}) = \log_2 16 = 4$

   PTS: 10 DIF: Average REF: Page 513 OBJ: 7-4.2 Subtracting Logarithms
   TOP: 7-4 Properties of Logarithms

7. ANS:
   14.7 in. Hg

$3.6 = -\left(\frac{100}{9}\right)\log\frac{P}{31}$	Substitute 3.6 for *h*.
$-\frac{9}{100}(3.6) = \log\frac{P}{31}$	Multiply both sides by $-\frac{9}{100}$.
$-0.324 = \log\frac{P}{31}$	Simplify.
$-0.324 = \log P - \log 31$	Apply the Quotient Property of Logarithms.
$-0.324 = \log P - 1.491$	Calculate.
$1.167 = \log P$	Add 1.491 to both sides.
$P = 10^{1.167}$	Apply the Inverse Property of Exponents and Logarithms.
$P \approx 14.7$	Calculate.

   PTS: 10 DIF: Average REF: Page 515 OBJ: 7-4.6 Application
   TOP: 7-4 Properties of Logarithms

8. ANS:
   $2\log_7 x$

$\log_7 x^3 - \log_7 x$	
$= 3\log_7 x - \log_7 x$	Use the Power Property of Logarithms.
$= 2\log_7 x$	Simplify.

   PTS: 10 DIF: Advanced TOP: 7-4 Properties of Logarithms

9. ANS:
$x = 12$

$\left(2^3\right)^{x+8}$	$= \left(2^5\right)^x$	Rewrite each side as powers of the same base.
$2^{3(x+8)}$	$= 2^{5x}$	To raise a power to a power, multiply the exponents.
$3(x+8)$	$= 5x$	The bases are the same, so the exponents must be equal.
x	$= 12$	

The solution is $x = 12$.

PTS: 10 DIF: Average REF: Page 522
OBJ: 7-5.1 Solving Exponential Equations
TOP: 7-5 Exponential and Logarithmic Equations and Inequalities

10. ANS:
19 years

$900 = 300(1.005)^{12t}$	Write 900 for the amount.
$3 = 1.005^{12t}$	Divide both sides by 300.
$\log 3 = \log 1.005^{12t}$	Take the log of both sides.
$\log 3 = (12t)\log 1.005$	Use the Power Property.
$\frac{\log 3}{12\log(1.005)} = t$	Divide by $12\log(1.005)$.
$t = 18.36$	Evaluate with a calculator.
$t \approx 19$ years	Round to the next year.

PTS: 10 DIF: Average REF: Page 523 OBJ: 7-5.2 Application
TOP: 7-5 Exponential and Logarithmic Equations and Inequalities

11. ANS:
$x = 5^{\frac{21}{4}}$

$\log_5 \dfrac{x^{10}}{x^6} = 21$	Apply the Quotient Property.
$\log_5 x^4 = 21$	Simplify.
$4\log_5 x = 21$	Use the Power Property.
$\log_5 x = \frac{21}{4}$	Divide.
$5^{\log_5 x} = 5^{\frac{21}{4}}$	Use 5 as the base for both sides.
$x = 5^{\frac{21}{4}}$	Use inverse properties.

PTS: 10 DIF: Advanced REF: Page 523
OBJ: 7-5.3 Solving Logarithmic Equations
TOP: 7-5 Exponential and Logarithmic Equations and Inequalities

12. ANS:
35 sec

$r = 335$ ft per sec	Find the constant of variation r.
$d = 335t$	Write the direct variation function.
$11{,}725 = 335t$	Substitute.
$t = 35$	Solve.

It would take 35 seconds for sound to travel 11,725 feet.

PTS: 10 DIF: Basic REF: Page 570
OBJ: 8-1.2 Solving Direct Variation Problems NAT: 12.5.4.c
TOP: 8-1 Variation Functions

13. ANS:
339.12 cm^3

$V = khr^2$	V varies jointly as h and r^2.
$157.00 = k(2)(25)$	Substitute 157.00 for V, 2 for h and 25 for r^2.
$3.14 = k$	Solve for k.
$V = (3.14)hr^2$	Replace k in the function.
$V = 3.14(3)(36)$	Substitute 3 for h and 36 for r^2.
$V = 339.12$	

PTS: 10 DIF: Average REF: Page 570
OBJ: 8-1.3 Solving Joint Variation Problems NAT: 12.5.4.c
TOP: 8-1 Variation Functions

14. ANS:
16 lawns
One method is to use $s_1 l_1 = s_2 l_2$.

$(6)(8) = (3)l_2$	Substitute given values.
$48 = 3l_2$	Simplify.
$16 = l_2$	Divide.

PTS: 10 DIF: Average REF: Page 571 OBJ: 8-1.5 Application
NAT: 12.5.1.e TOP: 8-1 Variation Functions

15. ANS:
1.25 atmospheres

$P = k\dfrac{T}{V}$	Write the original equation.
$1.6 = k\dfrac{280}{14}$	Substitute and solve for *k*.
$1.6 = 20k$	
$0.08 = k$	
$P = 0.08\dfrac{250}{16}$	Substitute for *k*, *T*, and *V*.
$P = 1.25$	

PTS: 10 DIF: Average REF: Page 572 OBJ: 8-1.7 Application
NAT: 12.5.1.e TOP: 8-1 Variation Functions

16. ANS:
$4y$; $y \neq 2$ or 0
Factor common factors out of the numerator and/or denominator. Divide out the common factors to simplify the expression. Finally, use the original denominator to determine any y-values for which the expression is undefined.

PTS: 10 DIF: Average REF: Page 577
OBJ: 8-2.1 Simplifying Rational Expressions NAT: 12.5.3.c
TOP: 8-2 Multiplying and Dividing Rational Expressions

17. ANS:
$6x^5z^3$

Arrange the expressions so like terms are together: $\dfrac{8 \cdot 9(x^4 \cdot x)(y^2 \cdot y^2)z^6}{3 \cdot 4 \cdot z^3 y^4}$.

Multiply the numerators and denominators, remembering to add exponents when multiplying: $\dfrac{72x^5y^4z^6}{12z^3y^4}$.

Divide, remembering to subtract exponents: $6x^5y^0z^3$.
Since $y^0 = 1$, this expression simplifies to $6x^5z^3$.

PTS: 10 DIF: Basic REF: Page 578
OBJ: 8-2.3 Multiplying Rational Expressions NAT: 12.5.3.c
TOP: 8-2 Multiplying and Dividing Rational Expressions

18. ANS:

$\frac{xy^8}{5}$

$\frac{5x^3}{3x^2y} \div \frac{25}{3y^9}$ — Rewrite as multiplication by the reciprocal.

$= \frac{5x^3}{3x^2y} \cdot \frac{3y^9}{25}$ — Simplify by canceling common factors.

$= \frac{xy^8}{5}$

PTS: 10 DIF: Basic REF: Page 579
OBJ: 8-2.4 Dividing Rational Expressions NAT: 12.5.3.c
TOP: 8-2 Multiplying and Dividing Rational Expressions

19. ANS:

There is no solution because the original equation is undefined at $x = 5$.

$\frac{x^2 + x - 30}{x - 5} = 11$ — Note that $x \neq 5$.

$\frac{(x-5)(x+6)}{x-5} = 11$ — Factor.

$x + 6 = 11$ — The factor $(x - 5)$ cancels.

$x = 5$

Because the left side of the original equation is undefined when $x = 5$, there is no solution.

PTS: 10 DIF: Average REF: Page 579
OBJ: 8-2.5 Solving Simple Rational Equations NAT: 12.5.3.c
TOP: 8-2 Multiplying and Dividing Rational Expressions

20. ANS:

$x + 2$

$A = lw$ — Area of a rectangle equals length times width.

$(x^2 + 10x + 16) = (x + 8)w$ — Substitute the area and length expressions given.

$\frac{(x+8)(x+2)}{(x+8)} = w$ — Factor and solve for w.

$(x + 2) = w$ — Simplify.

PTS: 10 DIF: Advanced NAT: 12.5.3.c
TOP: 8-2 Multiplying and Dividing Rational Expressions

21. ANS:

$\frac{x+5}{x+4}$

$\frac{x+6}{x-7}+\frac{-12x-59}{(x+4)(x-7)}$	Factor the denominators. The LCD is $(x+4)(x-7)$.
$=\left(\frac{x+4}{x+4}\right)\frac{x+6}{x-7}+\frac{-12x-59}{(x+4)(x-7)}$	Multiply by $\left(\frac{x+4}{x+4}\right)$.
$=\frac{x^2+10x+24}{(x+4)(x-7)}+\frac{-12x-59}{(x+4)(x-7)}$	
$=\frac{x^2-2x-35}{(x+4)(x-7)}$	Add the numerators.
$=\frac{(x+5)(x-7)}{(x+4)(x-7)}$	Factor the numerator.
$=\frac{x+5}{x+4}$	Divide the common factor.

PTS: 10 DIF: Average REF: Page 584 OBJ: 8-3.3 Adding Rational Expressions
NAT: 12.5.3.c TOP: 8-3 Adding and Subtracting Rational Expressions

22. ANS:

$\frac{x-6}{x-4}$

The expression is undefined at $x = 4$ and $x = -4$.

$\frac{2x^2-48}{(x-4)(x+4)} - \frac{x+6}{x+4}$	Factor the denominators.
$= \frac{2x^2-48}{(x-4)(x+4)} - \frac{x+6}{x+4}\left(\frac{x-4}{x-4}\right)$	The LCD is $(x-4)(x+4)$, so multiply $\frac{x+6}{x+4}$ by $\frac{x-4}{x-4}$.
$= \frac{2x^2-48-(x+6)(x-4)}{(x-4)(x+4)}$	Subtract the numerators.
$= \frac{2x^2-48-\left(x^2+2x-24\right)}{(x-4)(x+4)}$	Multiply the binomials in the numerator.
$= \frac{2x^2-48-x^2-2x+24}{(x-4)(x+4)}$	Distribute the negative sign.
$= \frac{x^2-2x-24}{(x-4)(x+4)}$	Write the numerator in standard form.
$= \frac{(x-6)(x+4)}{(x-4)(x+4)} = \frac{x-6}{x-4}$	Factor the numerator, and divide out common factors.

The expression is undefined at $x = 4$ and $x = -4$ because these values of x make the factors $(x-4)$ and $(x+4)$ equal 0.

PTS: 10 DIF: Average REF: Page 585
OBJ: 8-3.4 Subtracting Rational Expressions NAT: 12.5.3.c
TOP: 8-3 Adding and Subtracting Rational Expressions

23. ANS:
$x = 3$ or $x = 6$

$x(x) - 9(x) = -\frac{18}{x}(x)$	Multiply each term by the LCD.
$x^2 - 9x = -18$	Simplify. Note $x \neq 0$
$x^2 - 9x + 18 = 0$	Write in standard form.
$(x-3)(x-6) = 0$	Factor.
$x - 3 = 0$ or $x - 6 = 0$	Apply the Zero-Product Property.
$x = 3$ or $x = 6$	Solve for x.

Check:

$x - 9 = -\frac{18}{x}$

$3-9$	$-\frac{18}{3}$
-6	-6

$x - 9 = -\frac{18}{x}$

$6-9$	$-\frac{18}{6}$
-3	-3

PTS: 10 DIF: Average REF: Page 600 OBJ: 8-5.1 Solving Rational Equations
NAT: 12.5.4.a TOP: 8-5 Solving Rational Equations and Inequalities

24. ANS:
There is no solution.

$\frac{6x}{x-3}(x-3) = \frac{4x+6}{x-3}(x-3)$	Multiply each term by the LCD, $(x - 3)$.
$6x = 4x + 6$	Simplify. Note that x g 3.
$2x = 6$	Solve for x.
$x = 3$	

The solution $x = 3$ is extraneous because it makes the denominators of the original equation equal to 0. Therefore the equation has no solution.

PTS: 10 DIF: Average REF: Page 601 OBJ: 8-5.2 Extraneous Solutions
NAT: 12.5.4.a TOP: 8-5 Solving Rational Equations and Inequalities

25. ANS:
$4z^4$

$\sqrt[4]{256z^{16}}$

$= \sqrt[4]{256 \cdot z^4 \cdot z^4 \cdot z^4 \cdot z^4}$	Factor into perfect powers of four.
$= 4 \cdot z \cdot z \cdot z \cdot z$	Use the Product Property of Roots.
$= 4z^4$	Simplify.

PTS: 10 DIF: Basic REF: Page 611
OBJ: 8-6.2 Simplifying Radical Expressions NAT: 12.5.3.c
TOP: 8-6 Radical Expressions and Rational Exponents

26. ANS:
27

$(27)^{\frac{1}{3}} \cdot (27)^{\frac{2}{3}}$

$= (27)^{\frac{1+2}{3}}$ Product of Powers

$= (27)^1$ Simplify.

$= 27$

PTS: 10 DIF: Basic REF: Page 613
OBJ: 8-6.5 Simplifying Expressions with Rational Exponents NAT: 12.5.3.c
TOP: 8-6 Radical Expressions and Rational Exponents

27. ANS:
$x = 21$

$\sqrt{x-5} = 4$ Subtract –6 from both sides.

$x - 5 = 16$ Square both sides.

$x = 21$ Simplify.

Check

$-6 + \sqrt{21-5} = -2$

$-6 + \sqrt{16} = -2$

$-6 + 4 = -2$

$-2 = -2$ OK

PTS: 10 DIF: Basic REF: Page 628
OBJ: 8-8.1 Solving Equations Containing One Radical
TOP: 8-8 Solving Radical Equations and Inequalities

28. ANS:

$$\frac{y^2}{100}+\frac{x^2}{64}=1$$

Step 1 Choose the appropriate form of the equation.

$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$. Because the vertical axis is longer.

Step 2 Identify the values of *a* and *c*.
$a = 10$; The vertex (0, –10) gives the value of *a*.
$c = 6$; The focus (0, 6) gives the value of *c*.

Step 3 Use the equation $c^2 = a^2 - b^2$ to find the value of b^2.

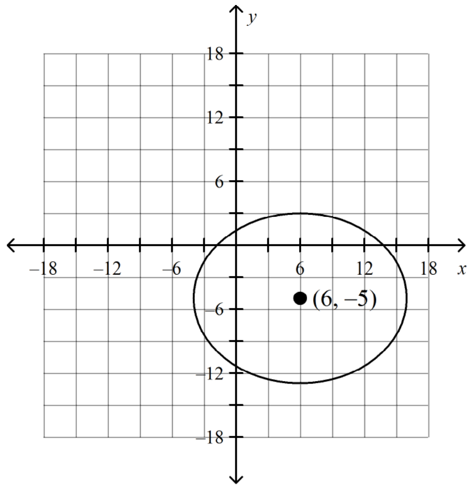
$$6^2 = 10^2 - b^2$$
$$64 = b^2$$

Step 4 Write the equation

$$\frac{y^2}{100}+\frac{x^2}{64}=1$$

PTS: 10 DIF: Average REF: Page 737
OBJ: 10-3.2 Using Standard Form to Write an Equation for an Ellipse
TOP: 10-3 Ellipses

29. ANS:



Step 1 Rewrite the equation as $\dfrac{(x-6)^2}{10^2}+\dfrac{(y+5)^2}{8^2}=1$

Step 2 Identify the values of h, k, a, and b.
$h=6$ and $k=-5$, so the center is $(6,\ -5)$.
$a=10$ and $b=8$; because $10>8$ the major axis is horizontal.

Step 3 The vertices are $(6\pm10,\ -5)$, or $(16,\ -5)$ and $(-4,\ -5)$,
and the co-vertices are $(6,\ -5\pm8)$, or $(6,\ 3)$ and $(6,\ -13)$.

PTS: 10 DIF: Basic REF: Page 738 OBJ: 10-3.3 Graphing Ellipses
TOP: 10-3 Ellipses

30. ANS:

$$\frac{x^2}{8000^2}+\frac{y^2}{6000^2}=1$$

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ Equation of an ellipse, with a horizontal major axis

$\dfrac{x^2}{8000^2}+\dfrac{y^2}{6000^2}=1$ Substitute 8000 for a and 6000 for b.

PTS: 10 DIF: Advanced TOP: 10-3 Ellipses

31. ANS:

$$\frac{y^2}{36} - \frac{x^2}{28} = 1$$

Step 1 The vertex and focus are on the vertical axis so the equation will be of the form:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.$$

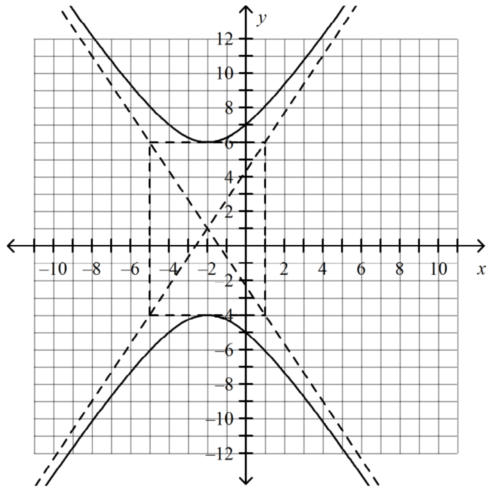
Step 2 The vertex is (0, 6) and the focus is (0, 8), so $a = 6$ and $c = 8$. Use $c^2 = a^2 + b^2$ to solve for b^2.

$8^2 = 6^2 + b^2$
$28 = b^2$

Step 3 The equation of the hyperbola is $\frac{y^2}{36} - \frac{x^2}{28} = 1$.

PTS: 10 DIF: Average REF: Page 745
OBJ: 10-4.2 Writing Equations of Hyperbolas TOP: 10-4 Hyperbolas

32. ANS:
Vertices: (–2, 6) and (–2, -4)
Co-vertices: (1, 1) and (–5, 1)
Asymptotes: $y-1=\frac{5}{3}(x+2)$ and $y-1=-\frac{5}{3}(x+2)$



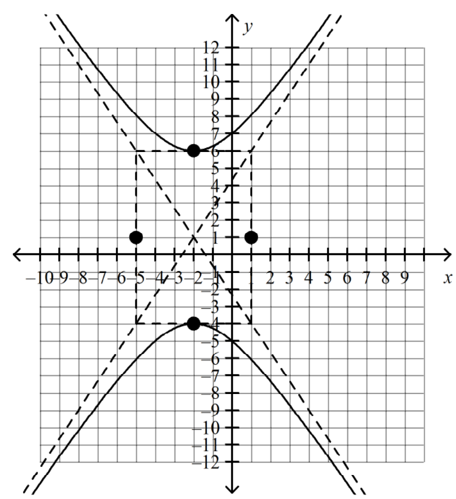
Step 1 The equation is of the form $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$, so the transverse axis is vertical with center (–2, 1).

Step 2 Because $a = 5$ and $b = 3$, the vertices are (–2, 6) and (–2, -4) and the co-vertices are (1, 1) and (–5, 1).

Step 3 The equations of the asymptotes are $y-1=\frac{5}{3}(x+2)$ and $y-1=-\frac{5}{3}(x+2)$.

Step 4 Draw a box using the vertices and co-vertices.Draw the asymptotes through the corners of the box.

Step 5 Draw the hyperbola using the vertices and the asymptotes.



PTS: 10 DIF: Average REF: Page 746 OBJ: 10-4.3 Graphing a Hyperbola
TOP: 10-4 Hyperbolas

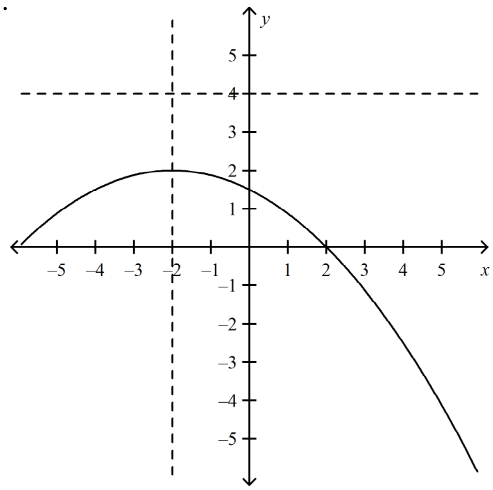
33. ANS:

$y = \frac{1}{56}x^2$

Step 1 Because the directrix is a horizontal line, the equation is in the form $y = \frac{1}{4p}x^2$. The vertex is above the directrix, so the graph will open upward.
Step 2 Because the directrix is $y = -14$, $p = 14$ and $4p = 56$.
Step 3 The equation of the parabola is $y = \frac{1}{56}x^2$.

PTS: 10 DIF: Average REF: Page 752
OBJ: 10-5.2 Writing Equations of Parabolas TOP: 10-5 Parabolas

34. ANS:
Vertex $(-2,2)$, focus $(-2,0)$, $p=-2$, axis of symmetry $x=-2$, and directrix $y=4$.



The standard form for a parabola with a vertical axis of symmetry is $y-k=\dfrac{1}{4p}(x-h)^2$.

Step 1 The vertex is (h, k) or $(-2,2)$.

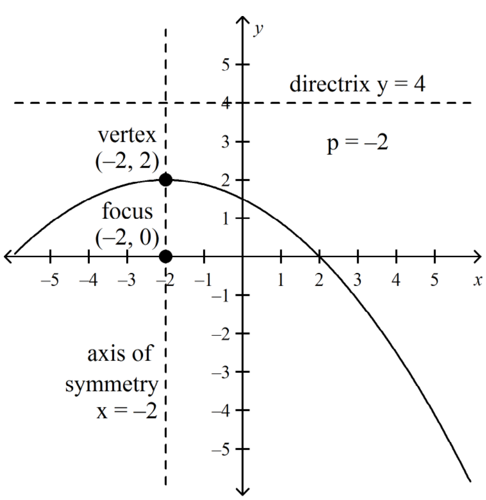
Step 2 $\dfrac{1}{4p}=-\dfrac{1}{8}$, so $4p=-8$ and $p=-2$.

Step 3 The graph has a vertical axis of symmetry, opens down, and $x=-2$.

Step 4 The focus is $(h, k+p)$. Substitute to get $(-2, 2+(-2))$ or $(-2,0)$.

Step 5 The directrix is a horizontal line $y=k-p$. Substitute to get $y=2-(-2)$ or $y=4$.

Step 6 Use the information to sketch the graph.



PTS: 10 DIF: Average REF: Page 753 OBJ: 10-5.3 Graphing Parabolas
TOP: 10-5 Parabolas

35. ANS:
ellipse
The standard form of an ellipse, where $a > b$, can be written as
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where the major axis is horizontal, or
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$, where the major axis is vertical.

$\frac{(x-2)^2}{3^2} + \frac{(y-4)^2}{7^2} = 1$ is an ellipse with a vertical axis.

PTS: 10 DIF: Advanced REF: Page 760
OBJ: 10-6.1 Identifying Conic Sections in Standard Form TOP: 10-6 Identifying Conic Sections

36. ANS:
hyperbola
The general form for a conic section is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

$A = 4, B = -5, C = -5$	Identify the values for A, B, and C.
$B^2 - 4AC = (-5)^2 - 4(4)(-5)$	Substitute into $B^2 - 4AC$.
$B^2 - 4AC = 105$	Simplify.

Because $B^2 - 4AC > 0$, the equation represents a hyperbola.

PTS: 10 DIF: Average REF: Page 761
OBJ: 10-6.2 Identifying Conic Sections in General Form TOP: 10-6 Identifying Conic Sections

37. ANS:
$x = 25\sqrt{2}$

$\sin\,\theta = \frac{\text{opp.}}{\text{hyp.}}$	You know the length of the hypotenuse and want to find the length of the side opposite the given angle. Use the sine function.
$\sin 45° = \frac{x}{50}$	Substitute 45° for θ, x for opp., and 50 for hyp.
$\frac{\sqrt{2}}{2} = \frac{x}{50}$	Substitute $\frac{\sqrt{2}}{2}$ for sin 45°.
$x = 25\sqrt{2}$	Multiply both sides by 50 to solve for x.

PTS: 10 DIF: Average REF: Page 930
OBJ: 13-1.2 Finding Side Lengths of Special Right Triangles NAT: 12.2.1.m
TOP: 13-1 Right-Angle Trigonometry

38. ANS:
56,713 ft

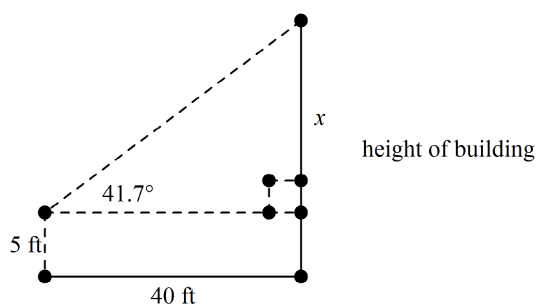
$\tan\,\theta = \dfrac{\text{opp.}}{\text{adj.}}$

$\tan 10° = \dfrac{10{,}000}{x}$	Substitute 10° for θ, 10,000 for opp., and x for adj.
$x(\tan 10°) = 10{,}000$	Multiply both sides by x.
$x = \dfrac{10{,}000}{\tan 10°} \approx 56{,}713$	Divide both sides by tan 10°. Use a calculator to simplify.

The land distance from the airplane to the airport is about 56,713 feet.

PTS: 10 DIF: Average REF: Page 930 OBJ: 13-1.3 Application
NAT: 12.2.1.m TOP: 13-1 Right-Angle Trigonometry

39. ANS:
41 ft
Step 1 Draw and label a diagram to represent the information given in the problem.



Step 2 Let x represent the height of the building from the surveyor's eye level. Determine the value of x.

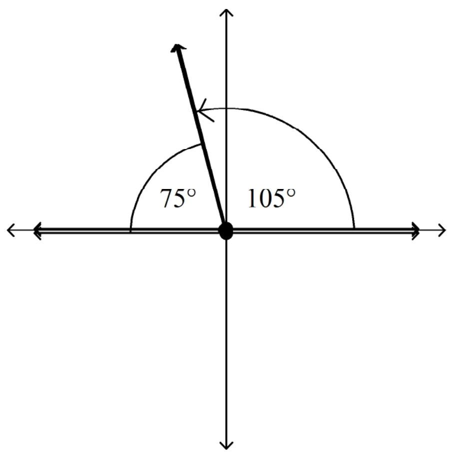
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	Use the tangent function.
$\tan 41.7 = \dfrac{x}{40}$	Substitute using x for opposite, 41.7 for θ, and 40 for adjacent.
$40(\tan 41.7) = x$	Multiply both sides by 40.
$x \approx 35.6387$	Use a calculator to solve for x.

Step 3 Determine the overall height of the building.

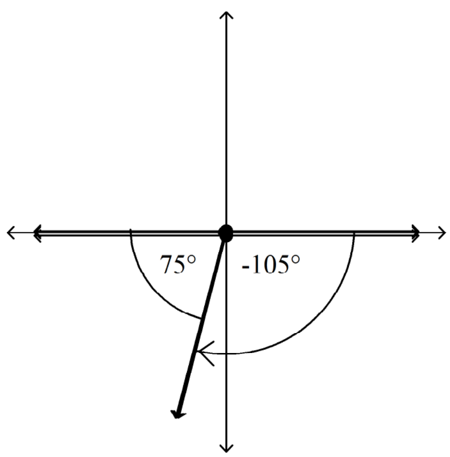
height $\approx x + 5 = 36 + 5$	The surveyor's eye level is 5 ft above the ground, so add 5 ft to the overall height of the building.
height ≈ 41	The height of the building is about 41 ft.

PTS: 10 DIF: Average REF: Page 931 OBJ: 13-1.4 Application
NAT: 12.2.1.m TOP: 13-1 Right-Angle Trigonometry

40. ANS:
38°
The reference angle is the acute angle created by the terminal side of θ and the x-axis.
For example:
When $\theta = 105°$, the reference angle measures 75°.



When $\theta = -105°$, the reference angle also measures 75°.



PTS: 10 DIF: Basic REF: Page 937 OBJ: 13-2.3 Finding Reference Angles
TOP: 13-2 Angles of Rotation

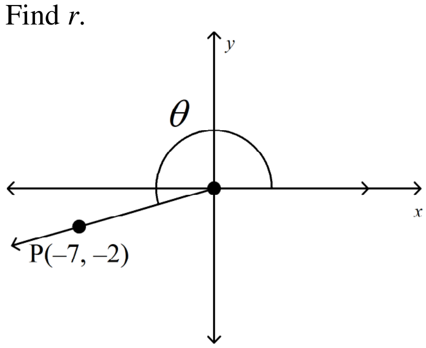
41. ANS:

$\sin\theta = -\frac{2\sqrt{53}}{53}$; $\csc\theta = -\frac{\sqrt{53}}{2}$;

$\cos\theta = -\frac{7\sqrt{53}}{53}$; $\sec\theta = -\frac{\sqrt{53}}{7}$;

$\tan\theta = \frac{2}{7}$; $\cot\theta = \frac{7}{2}$

Step 1 Plot point *P,* and use it to sketch angle θ in standard position.
Find *r*.



$r = \sqrt{(-7)^2 + (-2)^2} = \sqrt{53}$

Step 2 Find $\sin\theta$, $\cos\theta$, and $\tan\theta$.

$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{53}} = -\frac{2\sqrt{53}}{53}$; $\cos\theta = \frac{x}{r} = \frac{-7}{\sqrt{53}} = -\frac{7\sqrt{53}}{53}$; $\tan\theta = \frac{y}{x} = \frac{-2}{-7} = \frac{2}{7}$

Step 3 Use reciprocals to find $\csc\theta$, $\sec\theta$, and $\cot\theta$.

$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{53}}{2}$; $\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{53}}{7}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{7}{2}$

PTS: 10 DIF: Average REF: Page 938
OBJ: 13-2.4 Finding Values of Trigonometric Functions TOP: 13-2 Angles of Rotation

42. ANS:

$\frac{\sqrt{3}}{2}$

The angle passes through the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle.

$\cos\theta = x$

$\cos 30° = \frac{\sqrt{3}}{2}$

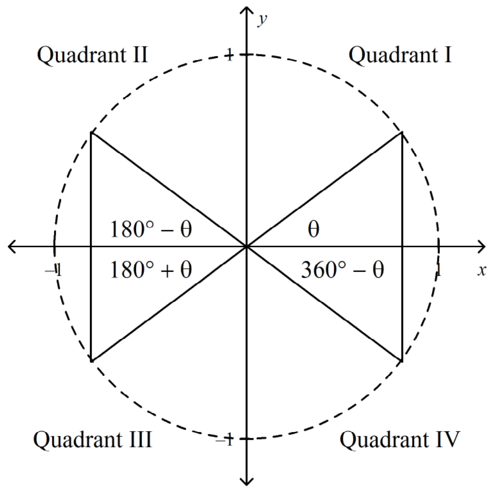
PTS: 10 DIF: Average REF: Page 944
OBJ: 13-3.2 Using the Unit Circle to Evaluate Trigonometric Functions
TOP: 13-3 The Unit Circle

43. ANS:
$-\frac{\sqrt{3}}{2}$

Step 1 Find the measure of the reference angle.
The angle is in Quadrant IV.
The measure of the reference angle is 60º.

Step 2 Find the sine of the reference angle.
$\sin 60^\circ = \frac{\sqrt{3}}{2}$

Step 3 Adjust the signs if needed.
In quadrant IV, the sine is negative.
$\sin 300^\circ = -\frac{\sqrt{3}}{2}$



PTS: 10 DIF: Average REF: Page 945
OBJ: 13-3.3 Using Reference Angles to Evaluate Trigonometric Functions
TOP: 13-3 The Unit Circle

44. ANS:
37°
Find the value of θ.

$\tan\theta = \frac{\text{opp.}}{\text{adj.}}$	Use the tangent ratio.
$\tan\theta = \frac{35}{46}$	Substitute 35 for opp. and 46 for adj. Then simplify.
$\theta = \tan^{-1}\left(\frac{35}{46}\right) = 37^\circ$	Use the inverse tangent function on your calculator to find the value of θ.

The angle of elevation of the sun from the end of the shadow is 37°.

PTS: 10 DIF: Basic REF: Page 952 OBJ: 13-4.3 Application
TOP: 13-4 Inverses of Trigonometric Functions

45. ANS:
$\theta = 162.5°$
Using the inverse sine function we get $\theta = \sin^{-1}(0.3) \approx 17.5°$.

Because $90° < \theta < 180°$ we need to find the angle in Quadrant II that has the same sine value as 17.5°.

$\theta \approx 180° - 17.5° = 162.5°$

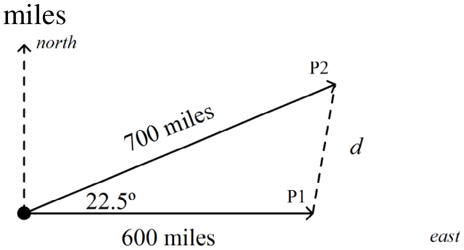
PTS: 10 DIF: Average REF: Page 952
OBJ: 13-4.4 Solving Trigonometric Equations
TOP: 13-4 Inverses of Trigonometric Functions

46. ANS:
23.0 ft^2

area $= \frac{1}{2}bh$	Write the area formula.
$h = a\sin C$	Solve for h using trigonometric ratios.
area $= \frac{1}{2}ab\sin C$	
area $= \frac{1}{2}(6)(8)\sin 73 = 23.0\text{ ft}^2$	Substitute.

PTS: 10 DIF: Basic REF: Page 958
OBJ: 13-5.1 Determining the Area of a Triangle TOP: 13-5 The Law of Sines

47. ANS:
272 miles



Each plane is flying for 2 hours, so they travel 600 and 700 miles respectively.

Use the law of cosines to find the distance, d.
$d^2 = (p1)^2 + (p2)^2 - 2(p1)(p2)\cos(D)$
$d^2 = (600)^2 + (700)^2 - 2(600)(700)\cos(22.5°) \approx 73{,}941.19$
$d \approx 272$ miles

PTS: 10 DIF: Average REF: Page 968
OBJ: 13-6.2 Problem-Solving Application TOP: 13-6 The Law of Cosines

48. ANS:
$14{,}464\text{ ft}^2$
Use Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$.
Find s.
$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(150+200+280) = 315$

Find the area of the triangle.
$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{315(315-150)(315-200)(315-280)}$

$A \approx 14{,}464\text{ ft}^2$

PTS: 10 DIF: Average REF: Page 969 OBJ: 13-6.3 Application
TOP: 13-6 The Law of Cosines